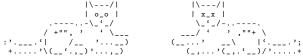
Quantum mechanics II, Chapter 2: Entanglement (Part 1)

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Sometimes observation kills.



Problem 1 : Bell States

Consider two spin-1/2 particles A and B with :

$$|\psi(A)\rangle = c_0^{(A)}|0\rangle_A + c_1^{(A)}|1\rangle_A$$

and

$$|\psi(B)\rangle = c_0^{(B)}|0\rangle_B + c_1^{(B)}|1\rangle_B.$$

The states $|0\rangle$ and $|1\rangle$ are the eigenstates of the $\hat{S}_z = \frac{\hbar}{2}\sigma_z$ operator with eigenvalues $+\hbar/2$ and $-\hbar/2$, respectively.

- 1. Write down the four possible basis states for the composite system $|\psi(A)\rangle \otimes |\psi(B)\rangle$ in terms of the basis vectors $\{|0\rangle_A, |1\rangle_A\}$ and $\{|0\rangle_B, |1\rangle_B\}$.
- 2. The Bell states are two-particle states given by:

$$|\Phi^{+}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{A} \otimes |0\rangle_{B} + |1\rangle_{A} \otimes |1\rangle_{B})$$

$$|\Phi^{-}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{A} \otimes |0\rangle_{B} - |1\rangle_{A} \otimes |1\rangle_{B})$$

$$|\Psi^{+}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{A} \otimes |1\rangle_{B} + |1\rangle_{A} \otimes |0\rangle_{B})$$

$$|\Psi^{-}\rangle = \frac{1}{\sqrt{2}} (|0\rangle_{A} \otimes |1\rangle_{B} - |1\rangle_{A} \otimes |0\rangle_{B}).$$

Show that these four Bell states form an orthonormal basis of the Hilbert space of the two spins $H = H_A \otimes H_B$.

- 3. Are the four Bell states separable?
- 4. We now consider the Bell state $|\Psi^{+}\rangle$.
 - (a) What is the probability of measuring $-\hbar/2$ when measuring the spin $\hat{S}_z^{(B)}$ of particle B?
 - (b) Let's assume now that the measurement of the spin B gives us $-\hbar/2$. What is the probability of subsequently measuring $+\hbar/2$ for the spin A?
- 5. We now consider $|\theta\rangle = \frac{1}{2}|\Phi^{+}\rangle + \frac{\sqrt{3}}{2}|\Psi^{-}\rangle$. Is this state correctly normalized? What is the probability of obtaining $\pm \hbar/2$ upon measuring $\hat{S}_{z}^{(A)}$ on the first spin?

6. What is the probability of obtaining $\pm \hbar/2$ upon measuring $\hat{S}_x^{(A)}$ (the x-axis spin of the first particle) for all of the four Bell states?

Problem 2: Composite system of two spin-1/2 particles

Consider the following Hamiltonian operator for two spin-1/2 particles:

$$\hat{H} = \mu_x \hat{S}_x^{(A)} \otimes \hat{S}_x^{(B)} + \mu_y \hat{S}_y^{(A)} \otimes \hat{S}_y^{(B)}$$

where $\hat{S}_x^{(A)}$ and $\hat{S}_y^{(A)}$ are the spin operators for the first spin and $\hat{S}_x^{(B)}$ and $\hat{S}_y^{(B)}$ are the spin operators for the second spin.

- 1. What are the conditions on the coefficients μ_x and μ_y such that \hat{H} is a valid observable?
- 2. Write down the matrix elements of \hat{H} in the basis of \hat{S}_z (i.e. $\{|0\rangle, |1\rangle\}^{\otimes 2}$).
- 3. Diagonalize the Hamiltonian in this basis and find its eigenvalues and the corresponding eigenvectors.
- 4. Are the eigenvectors of the Hamiltonian separable between the two particles?